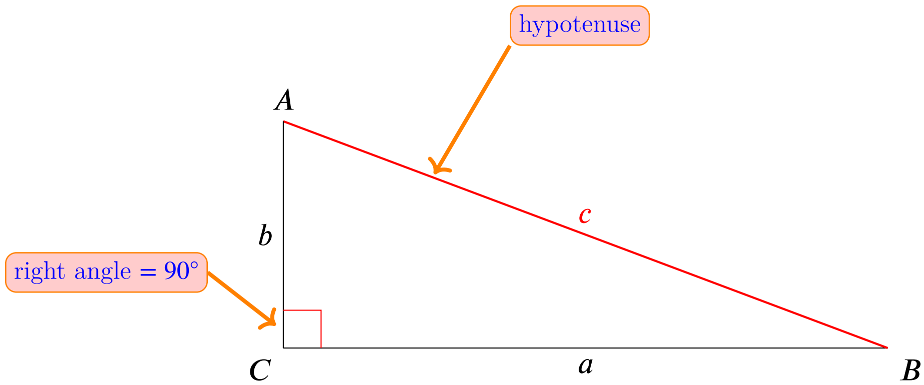
4.4. Pythagorean Theorem

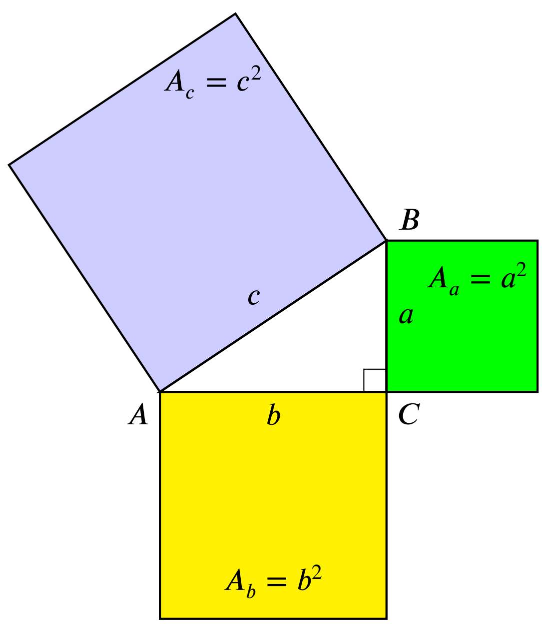
A **right-angle triangle** is a triangle containing a right angle (90°). A triangle cannot have more than one right angle, since the sum of the two right angles plus the third angle would exceed the 180° total possessed by a triangle. The side opposite the right angle is called the hypotenuse (side c in the figure below). The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).



Math notation for right-angle triangle

The Pythagorean Theorem states that the square of a hypotenuse is equal to the sum of the squares of the other two sides. It is one of the fundamental relations in Euclidean geometry.

$$c^2 = a^2 + b^2$$

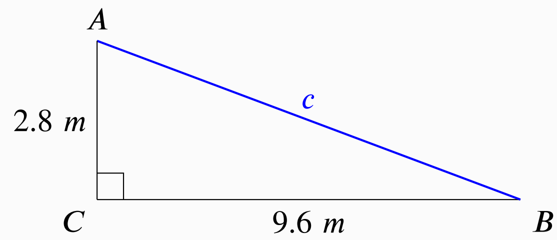


Pythagorean triangle with the squares of its sides and labels

Pythagorean triples are integer values of a, b, c satisfying this equation.

Finding the Sides of a Right Triangle

Example 1: Find the hypotenuse. 🖩



$$c^2 = a^2 + b^2 \quad \text{|substitute for } a \text{ and } b$$

$$c^2 = (9.6\ m)^2 + (2.8\ m)^2$$

$$c^2 = 92.16\ m^2 + 7.84\ m^2$$

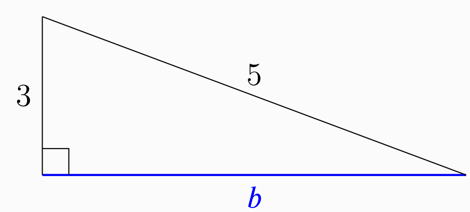
$$c^2 = 100\ m^2 \quad \text{|take the square root of each side}$$

$$\sqrt{c^2} = \sqrt{100\ m^2}$$

$$\sqrt{c^2} = \sqrt{100}\sqrt{m^2}$$

$$c = 10\ m$$

Example 2: Find the missing side b.



$$c^2 = a^2 + b^2 \quad \text{|subtract } a^2 \text{ from each side}$$

$$c^2 - a^2 = \cancel{a^2} + b^2 \cancel{- a^2}$$

$$c^2 - a^2 = b^2 \quad \text{|switch sides}$$

$$b^2 = c^2 - a^2 \quad \text{|substitute for } a \text{ and } c$$

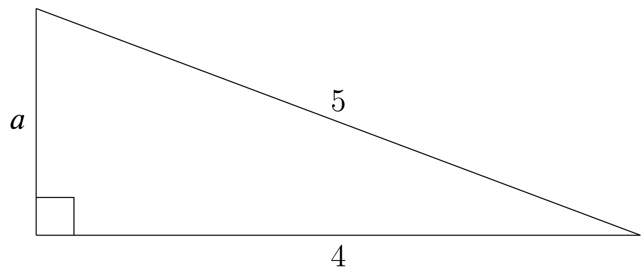
$$b^2 = 5^2 - 3^2$$

$$b^2 = 25 - 9 = 16 \quad \text{|take the square root of each side}$$

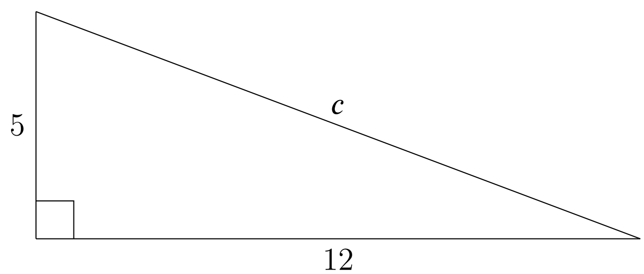
$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$

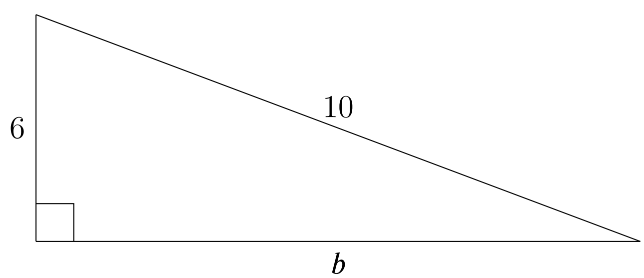
Practice 1: Find all the missing sides in each right angle triangle.



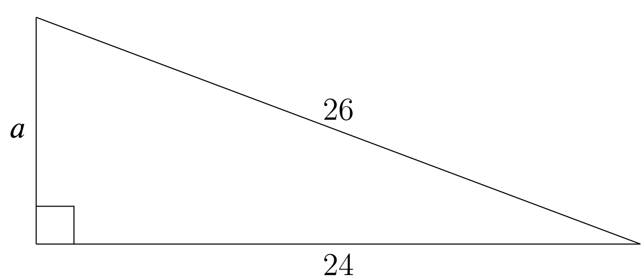
$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 5^2 - 4^2 = 25 - 16 = 9$$
$$\sqrt{a^2} = \sqrt{9}$$
$$a = 3$$



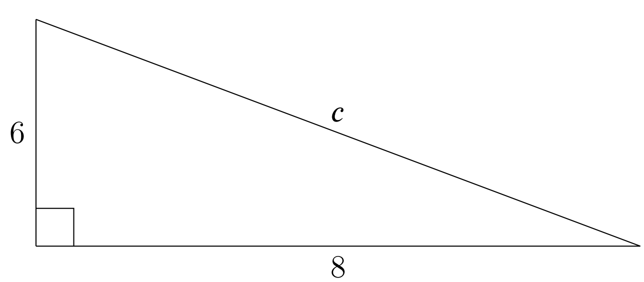
$$c^2 = a^2 + b^2$$
$$c^2 = 5^2 + 12^2 = 25 + 144 = 169$$
$$\sqrt{c^2} = \sqrt{169}$$
$$c = 13$$



$$c^2 = a^2 + b^2$$
$$b^2 = c^2 - a^2$$
$$b^2 = 10^2 - 6^2 = 100 - 36 = 64$$
$$\sqrt{b^2} = \sqrt{64}$$
$$b = 8$$

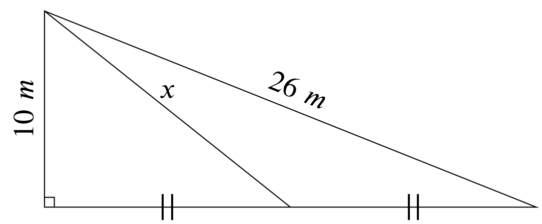


$$c^2 = a^2 + b^2$$
$$a^2 = c^2 - b^2$$
$$a^2 = 26^2 - 24^2 = 676 - 576 = 100$$
$$\sqrt{a^2} = \sqrt{100}$$
$$a = 10$$



$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2 = 36 + 64 = 100$$
$$\sqrt{c^2} = \sqrt{100}$$
$$c = 10$$

Challenge 1: Find x.



Use the Pythagorean theorem to find the base b in the larger triangle:

Given: hypotenuse : $c = 26\ m$ $a = 10\ m$ Find: $b = ?$

$a^2 + b^2 = c^2$ |subtract a^2 from both sides

$b^2 = (26\ m)^2 - (10\ m)^2$ |substitute values $c = 26\ m$ and $a = 10\ m$

$b^2 = 676\ m^2 - 10\ m^2$

$b^2 = 576\ m^2$

$\sqrt{b^2} = \sqrt{576\ m^2}$ |take the square root of each side

$b = 24\ m$

In the smaller triangle, the base (let's call it y) is half the base of the larger triangle.

Given: $y = \frac{1}{2}b = \frac{1}{2}(24\ m) = 12\ m$ $a = 10\ m$ Find: hypotenuse : $x = ?$

Use the Pythagorean theorem:

$c^2 = a^2 + b^2$ |substitute: $c = x$, $a = a$ and $b = y$

$x^2 = a^2 + y^2$ |substitute values: $a = 10\ m$ and $y = 12\ cm$

$x^2 = (10\ m)^2 + (12\ m)^2$

$x^2 = 100\ m^2 + 144\ m^2$

$x^2 = 244\ m^2$ |take the square root of each side

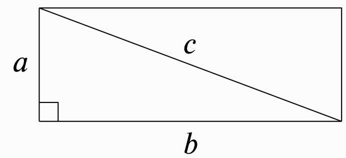
$\sqrt{x^2} = \sqrt{244\ m^2}$

$x = 15.6\ m$

Area of Right Triangle

The area of a triangle is equal to one half the base multiplied by the corresponding height: $A = \dfrac{bh}{2}$

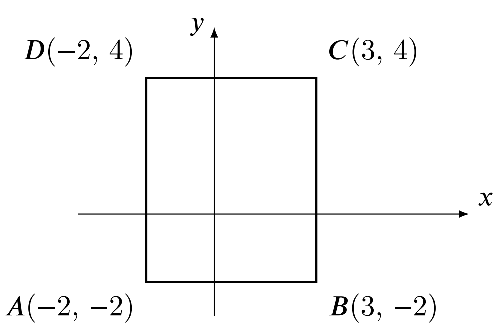
Example 3: Find the length of the diagonal of a rectangle that has width $a = 3$ and length $b = 4$.



The diagonal of a rectangle is the hypotenuse of a right-angle triangle. Use the Pythagorean Theorem.

$$c^2 = a^2 + b^2$$
$$c^2 = 3^2 + 4^2 = 9 + 16 = 25$$
$$\sqrt{c^2} = \sqrt{25}$$
$$c = 5$$ The length of the diagonal is $c = 5$.

Challenge 2: What is the length BD shown in the figure?



Points A, B, and D form the right-angle triangle with hypotenuse BD.
The length AB equals to change in x, from -2 in A($\underline{-2}$, -2) to 3 in B($\underline{3}$, -2), since points A and B have a fixed y coordinate.

$$AB = \Delta x = 3 - (-2) = 3 + 2 = 5$$

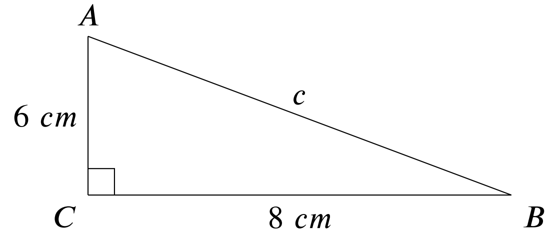
The length AD equals to change in y, from -2 in A(-2, $\underline{-2}$) to 4 in B(-2, $\underline{4}$), since points A and B have a fixed x coordinate.

$$AD = \Delta y = 4 - (-2) = 4 + 2 = 6$$

Use the Pythagorean theorem to find length BD, which is a hypotenuse of the right-angle triangle BAD.

$$BD^2 = 5^2 + 6^2 = 25 + 36$$
$$BD^2 = 61$$
$$\sqrt{BD^2} = \sqrt{61}$$
$$BD \approx 7.8$$

Challenge 3: Find the area of a square whose perimeter is the same as the perimeter of the triangle shown below.



To find the perimeter of the triangle we need to find the hypotenuse, side c, first.

$$P = a + b + c$$

The hypotenuse c of the triangle is

$$\begin{aligned} c^2 &= (6\ cm)^2 + (8\ cm)^2 \\ &= 36\ cm^2 + 64\ cm^2 \\ &= 100\ cm^2 \end{aligned}$$

Take square root of both sides

$$\sqrt{c^2} = \sqrt{100\ cm^2}$$
$$c = 10\ cm$$

The perimeter of triangle P is the sum of the lengths of all three sides of the triangle:

$$P = 6\ cm + 8\ cm + 10\ cm = 24\ cm$$

Perimeters of the triangle and the square are same. If the side of the square is a, then the perimeter of the square is

$$P = 4a$$

Divide by 4 and switch sides

$$a = \frac{P}{4}$$

Substitute value for P

$$a = \frac{24\ cm}{4} = 6\ cm$$

Finally, the area of the square is

$$A_S = a^2 = (6\ cm)^2 = 36\ cm^2$$