1.7. Long Multiplication and Division Review

Multiplication of large numbers is done using the distributive property of multiplication over addition. The numbers are first split into ones, tens, hundreds, etc.

$$259 \cdot 5 \ = \ (200 + 50 + 9) \cdot 5 \ = \ 200 \cdot 5 + 50 \cdot 5 + 9 \cdot 5$$

An **algorithm** known as **long multiplication** is used to simplify writing:

$$
\begin{array}{r}
5 \\
\times\ 2\ 5\ 9 \\
\hline
4\ 5 \\
2\ 5 \\
+\ 1\ 0 \\
\hline
1\ 2\ 9\ 5
\end{array}
$$

multiply the ones $5 \cdot 9$ ones

multiply the tens $5 \cdot 5$ tens

multiply the hundreds $5 \cdot 2$ hundreds

Example 1: Multiply integers: 67 and 8.

$$
\begin{array}{r}
6\ 7 \\
\times\ \ 8 \\
\hline
5\ 3\ 6
\end{array}
\qquad\qquad
\begin{array}{r}
8 \\
\times 6\ 7 \\
\hline
5\ 6 \\
4\ 8 \\
\hline
5\ 3\ 6
\end{array}
$$

Practice 1: Calculate:

$$
\begin{array}{r} 6\ 5 \\ \times\ \ 3 \\ \hline \end{array}
\qquad
\begin{array}{r} 2\ 4 \\ \times\ \ 4 \\ \hline \end{array}
\qquad
\begin{array}{r} 1\ 9 \\ \times\ \ 5 \\ \hline \end{array}
\qquad
\begin{array}{r} 3\ 6 \\ \times\ \ 5 \\ \hline \end{array}
\qquad
\begin{array}{r} 1\ 8 \\ \times\ \ 7 \\ \hline \end{array}
\qquad
\begin{array}{r} 2\ 3 \\ \times\ \ 6 \\ \hline \end{array}
$$

$$
\begin{array}{r} 2 \\ \times 5\ 6 \\ \hline \end{array}
\qquad
\begin{array}{r} 4 \\ \times 1\ 2 \\ \hline \end{array}
\qquad
\begin{array}{r} 3 \\ \times 9\ 9 \\ \hline \end{array}
\qquad
\begin{array}{r} 3 \\ \times 4\ 5 \\ \hline \end{array}
\qquad
\begin{array}{r} 3 \\ \times 6\ 1 \\ \hline \end{array}
\qquad
\begin{array}{r} 4 \\ \times 8\ 2 \\ \hline \end{array}
$$

Example 2: Multiply 619 and 7 using long multiplication.

$$
\begin{array}{r}
6\ \ 1\ \ 9 \\
\times \qquad 7 \\
\hline
4\ \ 3\ \ ^13\ \ ^63 \\
\end{array}
\qquad \text{multiply the ones} \quad 619 \cdot 7 \text{ ones}
$$

Start by multiplying ones $9 \cdot 7 = 63$. Write ones (3) at ones spot, and carry the 6.

Multiply tens and add the 6 that was carried over $1 \cdot 7 + 6 = 7 + 6 = 13$. Write tens (3) at the tens spot and carry the 1.

Multiply hundreds and add the 1 that was carried over $6 \cdot 7 + 1 = 42 + 1 = 43$. Write hundreds (3) at the hundreds spot and carry the 4.

There are no thousands in this problem, so just use the 4 that was carried over to the thousand column.

Practice 2: Calculate:

$$
\begin{array}{r}
2\ 1\ 5 \\
\times \quad 2 \\
\hline
\end{array}
\qquad
\begin{array}{r}
2\ 1\ 6 \\
\times \quad 2 \\
\hline
\end{array}
\qquad
\begin{array}{r}
3\ 7\ 0 \\
\times \quad 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
3\ 0\ 8 \\
\times \quad 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
4\ 5\ 0 \\
\times \quad 5 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
6\ 0\ 1 \\
\times \quad 9 \\
\hline
\end{array}
\qquad
\begin{array}{r}
4\ 3\ 0 \\
\times \quad 8 \\
\hline
\end{array}
\qquad
\begin{array}{r}
3\ 0\ 4 \\
\times \quad 5 \\
\hline
\end{array}
\qquad
\begin{array}{r}
5\ 0\ 4 \\
\times \quad 3 \\
\hline
\end{array}
\qquad
\begin{array}{r}
9\ 0\ 1 \\
\times \quad 9 \\
\hline
\end{array}
$$

Example 3: Multiply 342 and 37 using long multiplication.

$$
\begin{array}{r}
3\ 4\ 2 \\
\times \quad 3\ 7 \\
\hline
2\ 3\ 9\ 4 \\
+\ 1\ 0\ 2\ 6 \\
\hline
1\ 2\ 6\ 5\ 4
\end{array}
$$

multiply with ones $\quad 342 \cdot 7$ ones

multiply with tens $\quad 342 \cdot 3$ tens

Start by multiplying the first factor, 342, by the ones digit of the second factor, 7, and write the result underneath the line:

$$342 \cdot 7 \ = \ (2 + 40 + 300) \cdot 7 \ = \ 14 + 280 + 2100 \ = \ 2394$$

Multiply 342 by the number of tens of the second factor, 3. Write the result aligned with the tens column.

$$342 \cdot 3 \ = \ (2 + 40 + 300) \cdot 3 \ = \ 6 + 120 + 900 \ = \ 1026$$

Finally add the two rows in order to get the final result of 12654.

Practice 3: Multiply.

$$
\begin{array}{r}
1\ 6\ 4 \\
\times\quad 1\ 2 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
2\ 3\ 5 \\
\times\quad 1\ 5 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
7\ 0\ 5 \\
\times\quad 3\ 5 \\
\hline
\end{array}
$$

$$
\begin{array}{r}
2\ 1\ 4 \\
\times\quad 1\ 3 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
2\ 2\ 1 \\
\times\quad 6\ 4 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{r}
4\ 5\ 3 \\
\times\quad 2\ 3 \\
\hline
\end{array}
$$

Example 4: Divide $124 \div 4$ using long division.

$$
\begin{array}{r}
31 \\
4 \overline{)124} \\
12 \\
\hline
04 \\
4 \\
\hline
0
\end{array}
$$

Example 5: Divide $175 \div 7$ using long division.

$$
\begin{array}{r}
25 \\
7 \overline{)175} \\
14 \\
\hline
35 \\
35 \\
\hline
0
\end{array}
$$

Practice 4: Divide the following integers. Use long division.

a) $328 \div 8 =$

b) $295 \div 5 =$

c) $426 \div 6 =$

d) $576 \div 8 =$