1.7. Long Multiplication and Division Review

Multiplication of large numbers is done using the distributive property of multiplication over addition. The numbers are first split into ones, tens, hundreds, etc.

$$259 \cdot 5 \;=\; (200 + 50 + 9) \cdot 5 \;=\; 200 \cdot 5 + 50 \cdot 5 + 9 \cdot 5$$

An **algorithm** known as **long multiplication** is used to simplify writing:

$$\begin{array}{rrrr} & & & 5 \\ & \times & 2 & 5 & 9 \\ \hline & & & 4 & 5 \\ & & 2 & 5 & \\ + & 1 & 0 & & \\ \hline & 1 & 2 & 9 & 5 \end{array}$$

multiply the ones $5 \cdot 9$ ones

multiply the tens $5 \cdot 5$ tens

multiply the hundreds $5 \cdot 2$ hundreds

Example 1: Multiply integers: 67 and 8.

$$\begin{array}{r} 67 \\ \times\ 8 \\ \hline 536 \end{array} \qquad \begin{array}{r} 8 \\ \times 67 \\ \hline 56 \\ 48 \\ \hline 536 \end{array}$$

Practice 1: Calculate:

$$\begin{array}{r} 65 \\ \times\ 3 \\ \hline 195 \end{array} \quad \begin{array}{r} 24 \\ \times\ 4 \\ \hline 96 \end{array} \quad \begin{array}{r} 19 \\ \times\ 5 \\ \hline 95 \end{array} \quad \begin{array}{r} 36 \\ \times\ 5 \\ \hline 180 \end{array} \quad \begin{array}{r} 18 \\ \times\ 7 \\ \hline 126 \end{array} \quad \begin{array}{r} 23 \\ \times\ 6 \\ \hline 138 \end{array}$$

$$\begin{array}{r} 2 \\ \times 56 \\ \hline 12 \\ 10 \\ \hline 112 \end{array} \quad \begin{array}{r} 4 \\ \times 12 \\ \hline 8 \\ 4 \\ \hline 48 \end{array} \quad \begin{array}{r} 3 \\ \times 99 \\ \hline 27 \\ 27 \\ \hline 297 \end{array} \quad \begin{array}{r} 3 \\ \times 45 \\ \hline 15 \\ 12 \\ \hline 135 \end{array} \quad \begin{array}{r} 3 \\ \times 61 \\ \hline 3 \\ 18 \\ \hline 183 \end{array} \quad \begin{array}{r} 4 \\ \times 82 \\ \hline 8 \\ 32 \\ \hline 328 \end{array}$$

Example 2: Multiply 619 and 7 using long multiplication.

$$\begin{array}{r} 6\ 1\ 9 \\ \times\quad\ 7 \\ \hline 4\ 3\ {}^{1}3\ {}^{6}3 \end{array}$$

multiply the ones $619 \cdot 7$ ones

Start by multiplying ones $9 \cdot 7 = 63$. Write ones (3) at ones spot, and carry the 6.

Multiply tens and add the 6 that was carried over $1 \cdot 7 + 6 = 7 + 6 = 13$. Write tens (3) at the tens spot and carry the 1.

Multiply hundreds and add the 1 that was carried over $6 \cdot 7 + 1 = 42 + 1 = 43$. Write hundreds (3) at the hundreds spot and carry the 4.

There are no thousands in this problem, so just use the 4 that was carried over to the thousand column.

Practice 2: Calculate:

$$\begin{array}{r} 2\ 1\ 5 \\ \times\quad 2 \\ \hline 4\ 3\ 0 \end{array} \qquad \begin{array}{r} 2\ 1\ 6 \\ \times\quad 2 \\ \hline 4\ 3\ 2 \end{array} \qquad \begin{array}{r} 3\ 7\ 0 \\ \times\quad 3 \\ \hline 1\ 1\ 1\ 0 \end{array} \qquad \begin{array}{r} 3\ 0\ 8 \\ \times\quad 3 \\ \hline 9\ 2\ 4 \end{array} \qquad \begin{array}{r} 4\ 5\ 0 \\ \times\quad 5 \\ \hline 2\ 2\ 5\ 0 \end{array}$$

$$\begin{array}{r} 6\ 0\ 1 \\ \times\quad 9 \\ \hline 5\ 4\ 0\ 9 \end{array} \qquad \begin{array}{r} 4\ 3\ 0 \\ \times\quad 8 \\ \hline 3\ 4\ 4\ 0 \end{array} \qquad \begin{array}{r} 3\ 0\ 4 \\ \times\quad 5 \\ \hline 1\ 5\ 2\ 0 \end{array} \qquad \begin{array}{r} 5\ 0\ 4 \\ \times\quad 3 \\ \hline 1\ 5\ 1\ 2 \end{array} \qquad \begin{array}{r} 9\ 0\ 1 \\ \times\quad 9 \\ \hline 8\ 1\ 0\ 9 \end{array}$$

Example 3: Multiply 342 and 37 using long multiplication.

$$\begin{array}{rrrrr} & & 3 & 4 & 2 \\ & \times & & 3 & 7 \\ \hline & & 2 & 3 & 9 & 4 \\ + & 1 & 0 & 2 & 6 & \\ \hline & 1 & 2 & 6 & 5 & 4 \end{array}$$

multiply with ones $342 \cdot 7$ ones

multiply with tens $342 \cdot 3$ tens

Start by multiplying the first factor, 342, by the ones digit of the second factor, 7, and write the result underneath the line:

$$342 \cdot 7 \;=\; (2 + 40 + 300) \cdot 7 \;=\; 14 + 280 + 2100 \;=\; 2394$$

Multiply 342 by the number of tens of the second factor, 3. Write the result aligned with the tens column.

$$342 \cdot 3 \;=\; (2 + 40 + 300) \cdot 3 \;=\; 6 + 120 + 900 \;=\; 1026$$

Finally add the two rows in order to get the final result of 12654.

Practice 3: Multiply.

$$\begin{array}{r} 164 \\ \times\ 12 \\ \hline 328 \\ 164 \\ \hline 1968 \end{array} \qquad \begin{array}{r} 235 \\ \times\ 15 \\ \hline 1175 \\ 235 \\ \hline 3525 \end{array} \qquad \begin{array}{r} 705 \\ \times\ 35 \\ \hline 3525 \\ 2115 \\ \hline 24675 \end{array}$$

$$\begin{array}{r} 214 \\ \times\ 13 \\ \hline 642 \\ 214 \\ \hline 2782 \end{array} \qquad \begin{array}{r} 221 \\ \times\ 64 \\ \hline 884 \\ 1326 \\ \hline 14144 \end{array} \qquad \begin{array}{r} 453 \\ \times\ 23 \\ \hline 1359 \\ 906 \\ \hline 10419 \end{array}$$

Example 4: Divide $124 \div 4$ using long division.

$$\begin{array}{r} 31 \\ 4\overline{)124} \\ \underline{12} \\ 04 \\ \underline{4} \\ 0 \end{array}$$

Example 5: Divide $175 \div 7$ using long division.

$$\begin{array}{r} 25 \\ 7\overline{)175} \\ \underline{14} \\ 35 \\ \underline{35} \\ 0 \end{array}$$

Practice 4: Divide the following integers. Use long division.

a) $328 \div 8 = 41$

$$\begin{array}{r} 41 \\ 8\overline{)328} \\ \underline{32} \\ 08 \\ \underline{8} \\ 0 \end{array}$$

b) $295 \div 5 = 59$

$$\begin{array}{r} 59 \\ 5\overline{)295} \\ \underline{25} \\ 45 \\ \underline{45} \\ 0 \end{array}$$

c) $426 \div 6 = 71$

$$\begin{array}{r} 71 \\ 6\overline{)426} \\ \underline{42} \\ 06 \\ \underline{6} \\ 0 \end{array}$$

d) $576 \div 8 = 72$

$$\begin{array}{r} 72 \\ 8\overline{)576} \\ \underline{56} \\ 16 \\ \underline{16} \\ 0 \end{array}$$