Activity: Happy and Unhappy Numbers

There are numbers that are called **happy numbers**. Here is the algorithm **\sigma** used to find them:

- 1. Pick a number n you wish to test.
- 2. Let n_1 be the sum of the squares of the digits in number n.
- 3. Repeat step 2 until you reach: 0, 1, 4, 16, 20, 37, 42, 58, 89, or 145.
- 4. If you reached 1, the number you started with is happy!

Numbers that are not happy are said to be **sad** or **unhappy**.

Example 1: Let's check if number 97 is happy:

$$97 \longrightarrow 9^2 + 7^2 = 81 + 49 = 130$$

 $130 \longrightarrow 1^2 + 3^2 + 0^2 = 1 + 9 + 0 = 10$
 $10 \longrightarrow 1^2 + 0^2 = 1 + 0 = 1$

Number 97 is a happy number.

Problem 1: Check if the following numbers are happy or sad: 56, 556, 801, 937 and 961.

Challenge 1: The algorithm described above terminates when you reach 0, 1, 4, 16, 20, 37, 42, 58, 89, or 145. Why?

Hint: Look at each of the termination numbers and check what would happen if you keep going.

Challenge 2: Write a computer program to test if a number is happy or not. Use this program to find all happy numbers between 0 and 1001.