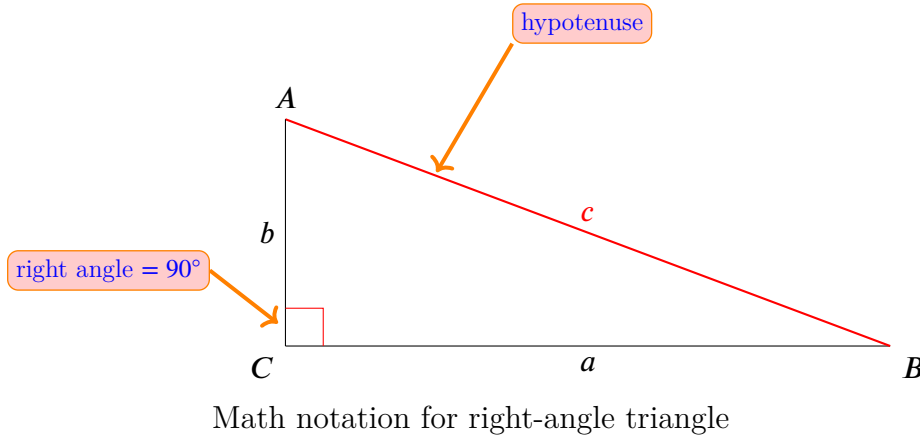


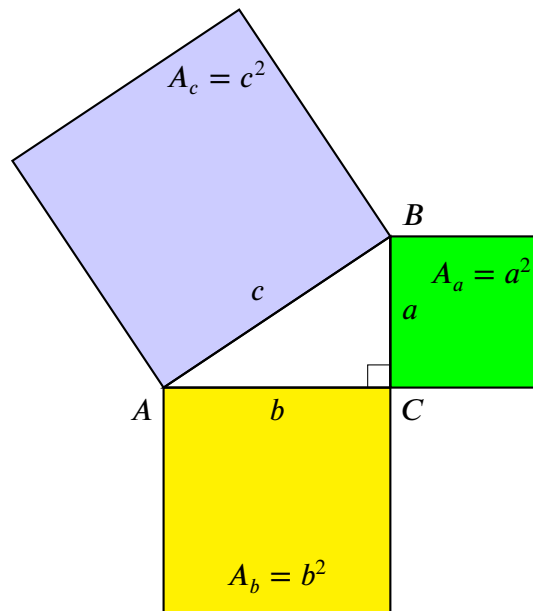
4.4. Pythagorean Theorem

A **right-angle triangle** is a triangle containing a right angle (90°). A triangle cannot have more than one right angle, since the sum of the two right angles plus the third angle would exceed the 180° total possessed by a triangle. The side opposite the right angle is called the hypotenuse (side c in the figure below). The sides adjacent to the right angle are called legs (or catheti, singular: cathetus).



The Pythagorean Theorem states that the square of a hypotenuse is equal to the sum of the squares of the other two sides. It is one of the fundamental relations in Euclidean geometry.

$$c^2 = a^2 + b^2$$

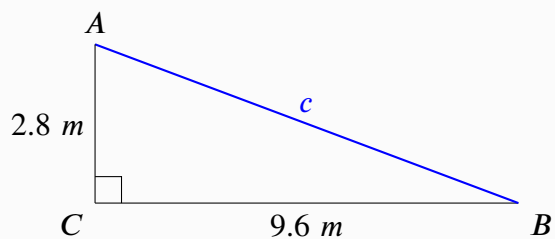


Pythagorean triangle with the squares of its sides and labels

Pythagorean triples are integer values of a , b , c satisfying this equation.

Finding the Sides of a Right Angled Triangle

Example 1: Find the hypotenuse. 🏠



$$c^2 = a^2 + b^2 \quad \text{|substitute for } a \text{ and } b$$

$$c^2 = (9.6 \text{ m})^2 + (2.8 \text{ m})^2$$

$$c^2 = 92.16 \text{ m}^2 + 7.84 \text{ m}^2$$

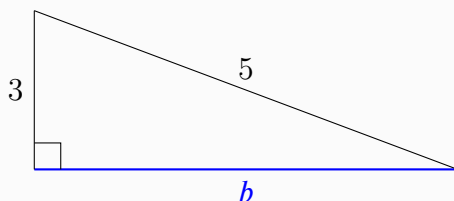
$$c^2 = 100 \text{ m}^2 \quad \text{|take the square root of each side}$$

$$\sqrt{c^2} = \sqrt{100 \text{ m}^2}$$

$$\sqrt{c^2} = \sqrt{100} \sqrt{\text{m}^2}$$

$$c = 10 \text{ m}$$

Example 2: Find the missing side b .



$$c^2 = a^2 + b^2 \quad \text{|subtract } a^2 \text{ from each side}$$

$$c^2 - a^2 = \cancel{a^2} + b^2 - \cancel{a^2}$$

$$c^2 - a^2 = b^2 \quad \text{|switch sides}$$

$$b^2 = c^2 - a^2 \quad \text{|substitute for } a \text{ and } c$$

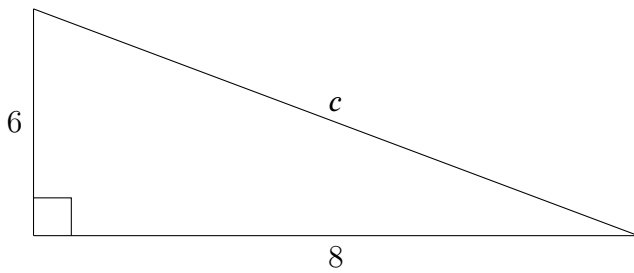
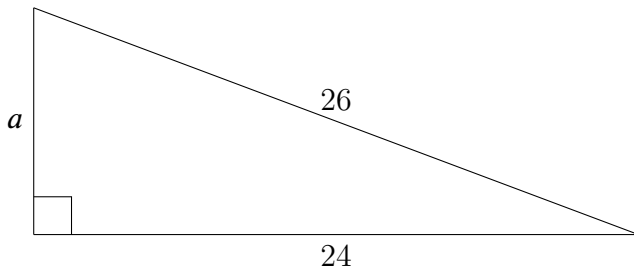
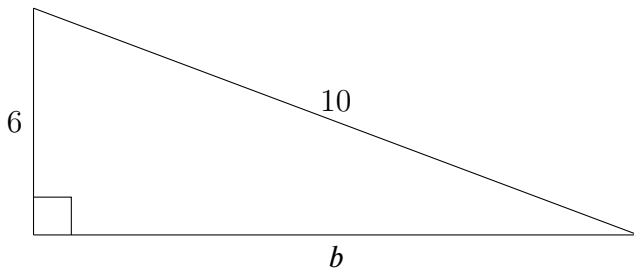
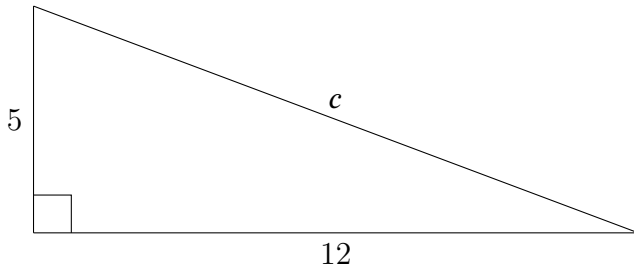
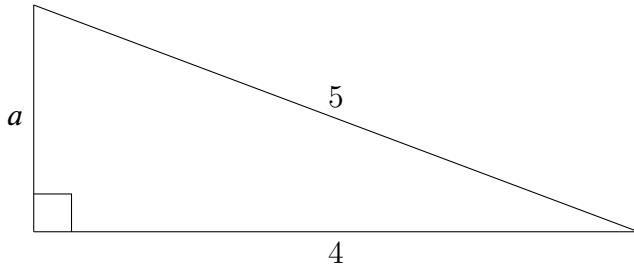
$$b^2 = 5^2 - 3^2$$

$$b^2 = 25 - 9 = 16 \quad \text{|take the square root of each side}$$

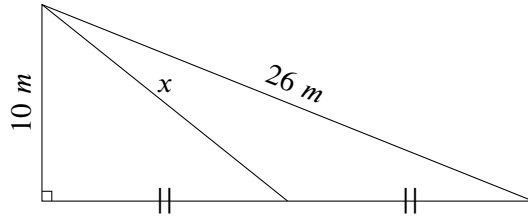
$$\sqrt{b^2} = \sqrt{16}$$

$$b = 4$$

Practice 1: Find all the missing sides in each right angled triangle.



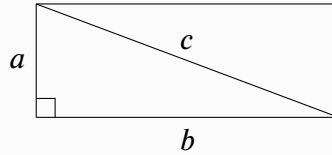
Challenge 1: Find x . 🧮 😊



Area of Right Angled Triangle

The area of a triangle is equal to one half the base multiplied by the corresponding height: $A = \frac{bh}{2}$

Example 3: Find the length of the diagonal of a rectangle that has width $a = 3$ and length $b = 4$.



The diagonal of a rectangle is the hypotenuse of a right-angle triangle. Use the Pythagorean Theorem.

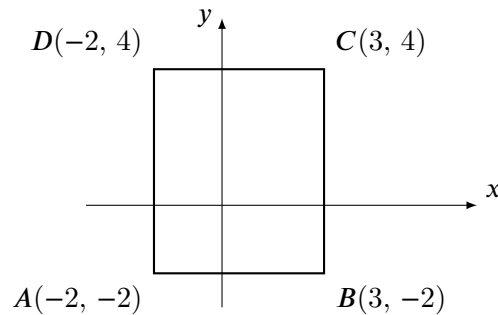
$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 4^2 = 9 + 16 = 25$$

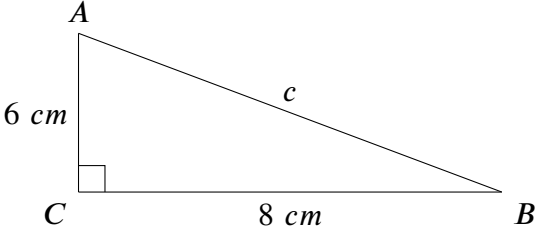
$$\sqrt{c^2} = \sqrt{25}$$

$$c = 5 \quad \text{The length of the diagonal is } c = 5.$$

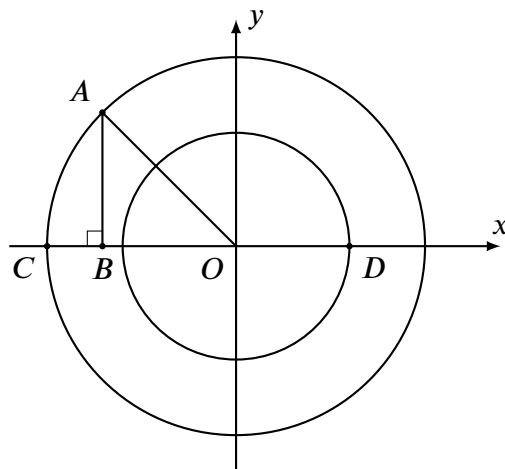
Challenge 2: What is the length BD shown in the figure? 🧮 🟢



Challenge 3: Find the area of a square whose perimeter is the same as the perimeter of the triangle shown below. 😊



Challenge 4: Two concentric circles with the center at the origin are shown below. $A(4, 3)$ is on the larger circle and CD is 9, what is the radius of the smaller circle? 🤔



Challenge 5: A rectangle with the area of $3\sqrt{2}$ is inscribed into a square with the side length of $a + b$, as shown below. Find the length of the rectangles' diagonal d , if $a : b = 2\sqrt{2} : 3$. 🤔

